$$|H(i\omega)| = \left(1 + \frac{\omega_-^2}{15\mathrm{Sc}^{2/3}}\right)^{1/4} \left(\frac{1 + 2.32\cdot 10^{-3}\omega_-^2}{1 + 4.45\cdot 10^{-2}\omega_-^2 + 1.49\cdot 10^{-4}\omega_-^4}\right)^{3/4},$$

where $\omega_- = \frac{\omega d}{k w_0}\left(\frac{\nu}{D}\right)^{1/3}$.

The phase of the FC is defined as the sum of the phases in Eqs. (16) and (18). The error in the resulting formulas does not exceed 1%. These can be used to obtain the spectra and the individual cases of turbulent velocity fluctuations.

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EXPERIMENTAL INVESTIGATION OF SONIC AND SUPERSONIC ANNULAR JETS

M. A. Koval' and A. I. Shvets

UDC 533.695.7

A considerable number of theoretical and experimental reports, which are surveyed in [1, 2], e.g., have been devoted to the investigation of flow in supersonic annular jets. The influence of the Mach number of the jet and the expansion ratio on the value of the base pressure has been established experimentally and the principal modes of flow in annular jets have been determined. Until now, however, the influence of the relative sizes of annular nozzles and of the profiling of the flow-through part on the flow has been little studied, in connection with which the present work was performed. Comparisons are made of the expansion ratio of the escaping jet, the wave structure, and the pressure in the base region.

The flow in three sonic and three supersonic jets escaping from annular nozzles with plane cuts was investigated experimentally. The subsonic flow channels in the nozzles provided for not less than fivefold compression of the stream and were profiled in such a way that a uniform stream was assured in the throats of the supersonic nozzles or at the cuts of the sonic nozzles. In the exit cross sections of the sonic nozzles the ratio of the inner to the outer diameter was $d/D = 0.5$, 0.75, and 0.9 (Fig. 1). The supersonic channels were conical, and the nozzles had the following parameters: $d/D = 0.6$; $\mu = 15°$; $\beta = 24°$; $M_a = 2.63$; $d/D = 0.68$; $\mu = 10°$; $\beta = 6°$; $M_a = 1.8$; $d/D = 0.91$; $\mu = 10°$; $\beta = 10°$; $M_a = 2.78$ (M_a is the rated Mach number of the nozzle).

1. Pressure in Base Region

In order to study the influence of the relative diameter of a nozzle on the relative base pressure $p_1 = p_{b0}/p_\infty$ measured at the axis of the nozzle face (the index b is for the base cut, 0 for the nozzle axis, and ∞ for parameters of the flooded space into which the jet discharges), in Fig. 1 we give the dependence $p_1 = f(n)$ for the three sonic jets and one of the supersonic ones ($M_a = 1$: $d/D = 0.9$ (1, 2), $d/D = 0.75$ (3, 4) $d/D = 0.5$ (5, 6); $M_a = 2.63$, $\mu = 15°$, $\beta = 24°$: $d/D = 0.6$ (7)). Since the expansion ratio of a sonic jet cannot be less than $n = 1$ ($n = p_a/p_\infty$, where p_a is the pressure at the nozzle cut), the quantity $n' = p_a M_a^2/p_\infty$ is taken as

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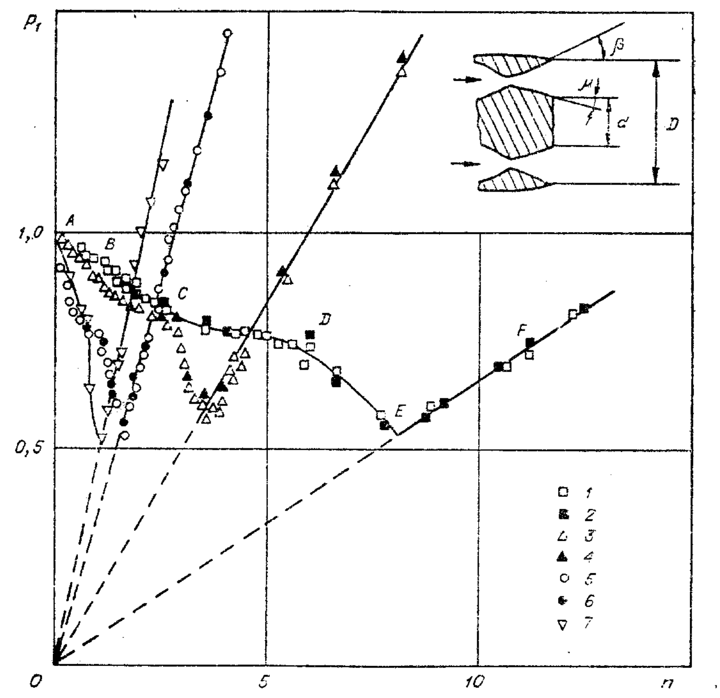

Fig. 1

the parameter uniquely characterizing the mode of subsonic flow. Henceforth it is assumed that $n = n'$ when $M_a \leq 1$. The pressure curves for the subsonic nozzles and for the supersonic nozzle with $M_a = 2.63$ are similar regardless of the ratio d/D. An exception is the dependence $p_1 = f(n)$ for the supersonic nozzle at small expansion ratios, i.e., in the mode of flow with separation in the nozzle.

Five characteristic modes having monotonic variation of the pressure are distinguished in the pressure curve $p_1 = f(n)$ for a sonic nozzle, which, as will be shown below, is connected with the flow structure in an annular jet. The boundaries of the sections of the modes, for the sonic nozzle with $d/D = 0.9$, for example, are marked in Fig. 1 by the points A, B, C, D, E, and F (A, B, C, D, E, and F are used arbitrarily below to designate the values of the expansion ratio for the corresponding modes of other nozzles). In section AB the jet is subsonic everywhere (point B corresponds to $n = 1$), while for an expansion ratio $n > 1$ the flow in the jet is supersonic. The bends in the pressure curve at points C, D, and E establish the boundaries for changes of the wave structure in the jet. For example, point D corresponds to the start of reorganization in the jet from an open to a closed base region while point E corresponds to the end of this reorganization, i.e., to completion of the formation of a sonic throat at the jet axis. We note that the reorganization of the flow in a jet from an open to a closed base region for sonic jets is not accomplished instantaneously but over the extent of some section DE. And this range of expansion ratios narrows with a decrease in the ratio d/D (Fig. 2, $n = f(d/D)$, $p_2 = p_{b0}/p_a$, p_a is the pressure at the nozzle cut, curve I is the start of the reorganization, and II is the end of the reorganization). For sonic jets the reorganization of the flow from an open to a closed base region always occurs at $n \geq 1$. For supersonic annular jets the reorganization of the flow from open to closed can occur both at $n \geq 1$ and at $n \leq 1$, depending on the Mach number M_a and the ratio d/D. An increase in d/D leads to an increase in the reorganization expansion ratio while an increase in the Mach number of the jet leads to a decrease in this expansion ratio [2]. In a mode of flow with a closed base region the pressure p_{b0} increases in proportion to the increase in n, i.e., in proportion to the increase in the pressure p_a.

The pressure over the face of an annular nozzle was measured in the experiment. Profiles of the pressure over the nozzle face at different expansion ratios for the sonic nozzle with $d/D = 0.9$ are presented in Fig. 3 ($p_3 = p_b/p_{b0}$ and $\bar{r} = r/(d/2)$ is the relative rounding radius of the nozzle face). It is seen that up to the start of the reorganization of flow in the jet from an open to a closed base region ($n \lesssim 6.7$) the pressure is constant over the base, but from $n \gtrsim 6.7$ up to an expansion ratio $n = 10.6$ (the expansion ratio for the end of reorganization of the base region is $n_E = 8$) the relative pressure over the base decreases toward the inner edge of the nozzle. Then at $n \geq 10.6$ the pressure is equalized over the nozzle base and at $n \gtrsim 11.2$ it hardly depends on the expansion ratio. For example, tests with $n = 12.3$, 79, and 97 gave the same pressure distribution as for $n = 11.2$. Thus, stabilization of the flow in a closed jet sets in at an expansion ratio greater than the expansion ratio corresponding to the end of a closed base region.

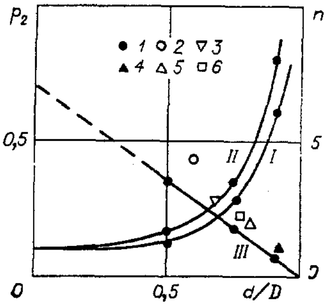

Fig. 2

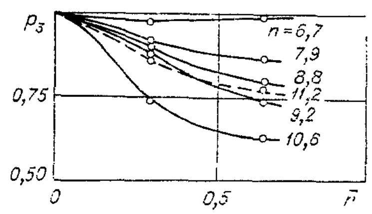

Fig. 3

The proportionality of the pressure variation to the expansion ratio for a closed jet means that in the case of a mode of flow with a closed base region the ratio $p_2 = p_{b0}/p_a$ for each concrete nozzle is a constant whose value is mainly determined by the nozzle geometry. For sonic nozzles the ratio p_2 depends linearly on d/D and does not depend on n (see Fig. 2, points 1 for $p_2 = f(d/D)$, line III). Extending the line III to the section of d/D = 0–0.5 gives a value of $p_2 \simeq 0.7$ at d/D = 0, which corresponds approximately to the ratio p_b/p_a (where p_a is the pressure at the cylinder near the rim) for a cylinder with a flat rear face when a sonic stream flows over it longitudinally [2].

In the case of the external problem of flow over bodies the relative base pressure p_2 decreases with an increase in the Mach number of the undisturbed stream, with the exception of a narrow section of transonic velocities where a slight pressure increase is observed, and therefore one can assume that for supersonic annular jets p_2 must be less than this value for a sonic jet at the same d/D. Under the experimental conditions, however, the points p_2 for supersonic jets are located above the curve for a sonic jet (see Fig. 2; points 2 correspond to $M_a = 2.63$, $\mu = 15°$, $\beta = 24°$; 3 to $M_a = 1.8$, $\mu = 10°$, $\beta = 6°$; 4 to $M_a = 2.78$, $\mu = 10°$, $\beta = 10°$; 5 to $M_a = 1.99$, $\mu = 10°$, $\beta = 10°$ [3]). This is due to the fact that in the experiment we did not use Laval nozzles with a uniform velocity field at the cut but conical nozzles with bending of the stream by angles μ and β. It is known that narrowing the rear ends of streamlined bodies increases the value of the base pressure [2]. In annular nozzles the analog of the bevel of the rear end is the angle of inclination μ of the inner shell to the axis. The angle of inclination β of the outer shell also affects the quantity p_2, since the experimental value of p_2 for a supersonic jet with $M_a = 2.54$, $\mu = 0$, and $\beta = 8°$ [4] (see Fig. 2, points 6) is also located above the line III for a sonic jet.

2. Stream Structure in Annular Jets

Since the structure of a supersonic annular jet has been adequately studied [1, 2], below principal attention is paid to the structure of sonic annular jets. It is interesting to establish the connections between the flow structure in a jet, the base pressure p_{b0}, and the expansion ratio (see Fig. 1). At subcritical pressures p_{0j} in the forechamber of the model ($n \leq 1$) a subsonic annular jet forms at the nozzle cut; its outer diameter decreases up to the region of joining of the annular jet into a solid jet at a distance of (1–1.5)D from the nozzle cut and then it increases again (Fig. 4, diagrams constructed from schlieren photographs of the flow; 1 are shock waves and 2 are boundaries of the jet). The minimum relative diameter H/D of a closed subsonic jet grows with a decrease in d/D: $H/D \simeq 0.5$ for d/D = 0.9, $H/D \simeq 0.7$–0.8 for d/D = 0.75, and H/D = 0.91–1 for d/D = 0.5 (the measurements of H correspond to an expansion ratio $n' = n \approx 0.8$). With an increase in the expansion ratio the pressure p_{b0} decreases monotonically in the mode of flow with a subsonic velocity. At point B (see Fig. 1) the form of the pressure curve changes somewhat with the presence of a slight bend, and at $n > n_B = 1$ a system of compression shocks is created in the jet. The formation of the wave structure is essentially influenced by the relative dimensions of the nozzles, i.e., by d/D. For thin annular jets (d/D = 0.9) within the limits of small expansion ratios the shape of the shock waves is close to the shape of the shock waves of plane jets with the corresponding expansion ratios. But with a decrease in d/D the shape of the shock waves begins to be affected by the axial symmetry in addition to the influence of the decreased pressure in the base region, as a result of which the shock waves are deflected toward the axis.

It has been observed experimentally that for both supersonic and sonic jets the first two to four cells formed by the shock waves of the jet are relatively stable. The subsequent cells undergo marked oscillations accompenied by irregular reorganizations of the Mach waves and regular interactions of the waves at the nodes. In the region of joining of the jet into a solid jet the shock-wave structure of the jet in schlieren photographs becomes poorly distinguishable.

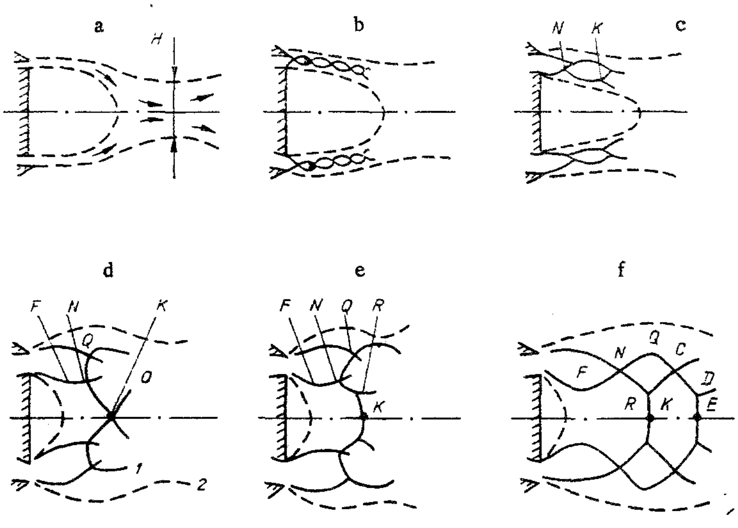

Fig. 4

As the point C is approached (see Fig. 1) the largest number of cells occurs in the jet, clearly visible in the photographs, and the thinner the annular jet ($d/D \to 1$), the larger the number of these cells. At point C on the pressure curve $p_1 = f(n)$ a slight bend forms, while the monotonic nature of the pressure curve changes again at $n \geq n_C$. In this case the cells, except for the first and second, start to break up, and by the time an expansion ratio n_D occurs (see Fig. 1) only the first two cells are clearly distinguished in the jet (Fig. 4c).

An increase in the expansion ratio $n > n_D$ is accompanied by intensive reorganization of the wave structure of the jet and by a strong decrease in the pressure p_1 (see Fig. 1, section DE). Reorganization of the flow in the base region from an open to a closed base region occurs in section DE. The transition to an expansion ratio n_E leads to closing of the base region of an annular jet with sonic and supersonic flow at the axis. In section DE the hanging wave NK (Fig. 4c) of the second cell becomes unstable with an increase in the expansion ratio and is smeared out on the photographs near point N, approaches the axis, and by the time an expansion ratio n_E occurs the wave NK becomes almost plane, forming a kind of Mach disk.

At $n > n_E$ the plane wave NK changes into a nearly conical wave which is regularly reflected from the axis as wave KO (Fig. 4d). Such behavior of wave NK ($n > n_E$), regardless of the expansion ratio, was only observed for the jet with $d/D = 0.9$. An almost conical wave did not exist in jets with $d/D = 0.75$ and 0.5. In these jets the plane wave NK is reorganized into a bridge-shaped wave NRK with an increase in the expansion ratio $n > n_E$ (Fig. 4e).

An increase in the expansion ratio ($n > n_E$) is accompanied by a decrease in the wave NQ until the points N and Q merge (Fig. 4e), forming a regular wave reflection at the point R (Fig. 4f), while the bridge-shaped wave RN is considerably removed from the nozzle cut in this case. The wave RC created at the point R is reflected from the wave QC and again forms ogival and bridge-shaped compression waves DE. Further downstream the wave structure of the jet is repeated and gradually smeared out, undergoing some oscillations.

The independence of p_2 from the expansion ratio and the occurrence of shock waves intersecting the flow axis beyond the base region (Fig. 4d-f) indicate the reorganization of the flow from an open to a closed base region. In a closed jet the inner hanging wave of the first barrel at point F (Fig. 4d, e) can curve considerably, depending on the expansion ratio.

However, the position of point F depends mainly on the ratio d/D (Fig. 5, $m/d = f(d/D)$; points 1 correspond to sonic jets in the mode at point E of Fig. 1). An increase in the expansion ratio $n > n_E$, accompanied by a decrease in pressure away from the axis of the face (see Fig. 3), leads to a decrease in the values of l and m. The decrease in l and m proceeds until those expansion ratios when the pressure at the face starts to level out somewhat. The minimum values of l and m are shown in Fig. 5 (points 2). With a further increase in the expansion ratio, however, the values of l and m increase again to the initial values (Fig. 5, points 1) and then no longer depend on the expansion ratio.

Since l and m indirectly reflect the size of the throat of the near wake, determined from the viscous boundary of the jet, and its position relative to the plane of the nozzle cut, and with allowance for the fact that

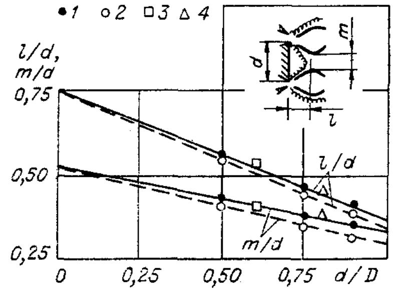

Fig. 5

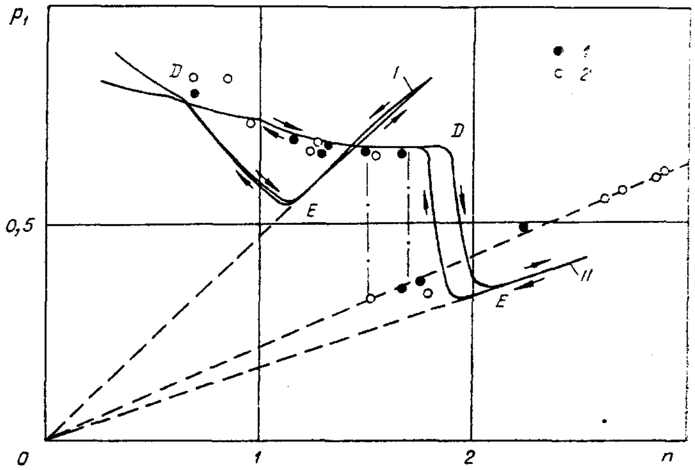

Fig. 6

in an unbounded stream the throat of the near wake decreases and approaches the base with an increase in the Mach number of the undisturbed stream [2], one would expect that l and m would have smaller values for supersonic jets than for sonic jets with the same ratio of diameters. However, the values of l and m (Fig. 5, points 3 and 4, respectively) for supersonic jets ($M_a = 2.63$, $\mu = 15°$, $\beta = 24°$; $M_a = 2.78$, $\mu = 10°$, $\beta = 10°$) practically coincide with the values of l and m for sonic jets. As is seen, an increase in the angle μ promotes not only an increase in the pressure p_{b0} but also an increase in the relative sizes l and m.

The movement of the point F on the inner hanging wave of the first cell (Fig. 4d, e), shown in Fig. 5 by the functions of l/d and m/d on d/D, qualitatively reflects the variation in the size of the throat of the near wake and of its location relative to the nozzle cut. Consequently, the ratios of the diameter of the wake throat and of its distance from the nozzle cut to the diameter of the inner shell of the nozzle grow with a decrease in d/D. An increase in the angle μ also leads to an increase in these parameters of the throat of the near wake.

3. On Hysteresis in Annular Nozzles

It is known that in annular jets the transitional modes of flow from an open to a closed base region and vice versa are accompanied by hysteresis phenomena [2, 4]. Hysteresis was not detected for the sonic jets in the investigated range of d/D. In Fig. 1 the values of p_1 are indicated for sonic jets (points 1, 3, 5 correspond to d/D = 0.9, 0.75, 0.5), obtained during an increase in the expansion ratio n and (points 2, 4, 6) during a decrease in n.

Consequently, hysteresis phenomena occur only for supersonic jets. Even in supersonic jets hysteresis phenomena do not always appear, however. For example, hysteresis is practically absent for the supersonic jet with $M_a = 2.63$, d/D = 0.6, $\mu = 15°$, and $\beta = 24°$, judging from the oscillogram (Fig. 6, curve I). But for the jet with $M_a = 2.78$, d/D = 0.91, $\mu = 10°$, and $\beta = 10°$ (Fig. 6, curve II) hysteresis occupies a considerable range of expansion ratios. If we construct the ratio $\delta = (n_E - n_D)/n_E$, then for sonic jets $\delta = 0.22$–0.25 (δ grows somewhat with an increase in d/D), $\delta = 0.36$ for a jet with $M_a = 2.63$, and $\delta = 0.1$ for $M_a = 2.78$. In comparing the values of δ for these jets we see that hysteresis is observed at small values of δ. Since a change in d/D has little effect on δ (e.g., for sonic jets with d/D = 0.5–0.9, $\delta = 0.22$–0.25), a comparison of the flow in jets with close Mach numbers M_a = 2.63 and 2.78 and with different nozzle geometries shows that with an increase in the angle μ (see Fig. 1), i.e., with an increase in δ, the hysteresis phenomena decrease and disappear. Conversely, a decrease in the angle μ promotes the development of hysteresis. For example, for a jet with $M_a = 2.54$, $d/D = 0.753$, $\mu = 0$, and $\beta = 8°$ [4] (Fig. 6; points 1 correspond to an increase in n, and points 2 to a decrease in n) the hysteresis zone has the maximum range of expansion ratios. Thus, hysteresis phenomena in supersonic jets essentially depend on the Mach number and the profiling of the nozzles.

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AERODYNAMIC FORCES ACTING ON THE BLADES OF A THREE-DIMENSIONAL ANNULAR ARRAY WITH NONSTEADY FLOW

V. P. Ryabchenko

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We present a computer realization of the solution of the three-dimensional problem of the nonsteady flow over the vane crown of an axial turbine by an irrotational stream of an ideal incompressible fluid, based on the vortex theory of a screw [1] and of a wing of finite span [2].

To solve this problem in [3, 4] the geometry of a blade crown was modeled by a straight three-dimensional array of plates enclosed between two planes, while in [5, 6] it was modeled by an annular array of vanes consisting of parts of helical surfaces. In the present report we adopt the second model, which evidently better describes the geometry of an actual turbine.

Because of the complexity of the algorithm suggested in [4–6], there are only individual examples of the calculation of nonsteady aerodynamic characteristics. Below, on the basis of a simple algorithm which is a generalization of the working method of [7] for an established flow, we analyze the influence of the three-dimensionality of the flow on the nonsteady aerodynamic forces acting on the vanes of a round array in a wide range of variation of the parameters of the array.

1. Let us consider a uniform stream of an ideal incompressible fluid with an axial velocity v through one array of vanes which are rotating with a constant angular velocity ω in a coaxial cylindrical channel which is infinite in the axial direction. We assume that the vanes can undergo synchronous, steady, harmonic vibrations of low amplitude at a frequency ω_1 and a constant phase shift $\mu\pi$ ($\mu = 2\sigma/N$, where $\sigma = 0, \pm1, \pm2, \ldots$; N is the number of vanes in the array).

We introduce cartesian (x, y, z) and cylindrical (x, r^*, θ^*) coordinate systems connected with the rotating vane array. The x axis is directed along the axis of rotation while the y and z axes are drawn in the plane perpendicular to it. The r^* and θ^* coordinates are connected with y and z by the usual equations, $y = r^* \cos\theta^*$ and $z = r^* \sin\theta^*$, where the angle θ^* is reckoned in the positive direction from the y axis (Fig. 1).

We assume that the vanes Σ_n ($n = 0, \ldots, N-1$) are infinitely thin and in the central position they consist of parts of helical surfaces bounded in the (r^*, θ^*) plane by a rectangle $\{r_1 \le r^* \le r_2,\ \alpha_n - \psi \le \theta^* \le \alpha_n + \psi\}$. Here $\alpha_n = 2\pi n/N$; n is the number of vanes; r_1 and r_2 are the radii of the inner and outer cylin-

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